



Application of Computational Limit Analysis & Design in the Middle East

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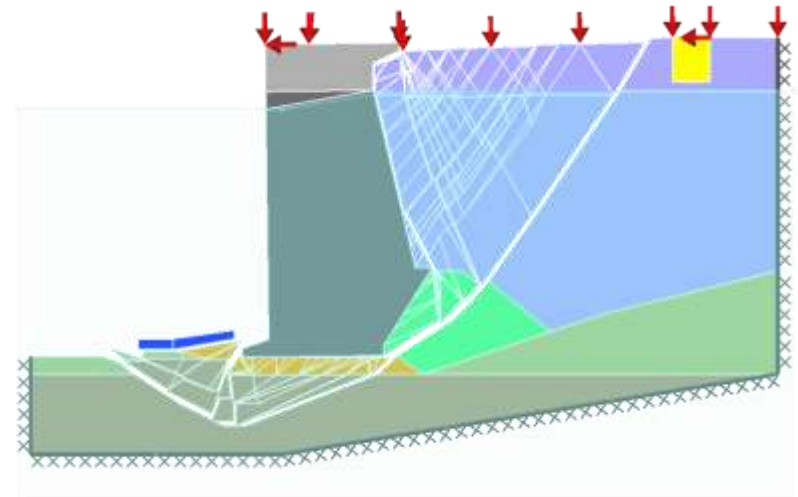
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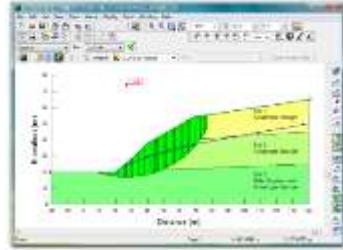
1. Background
2. Discontinuity Layout Optimization (DLO)
3. Application in design
4. Middle East examples
5. Conclusions



Background

Available geotechnical software

‘Traditional’:
based on hand
analysis
solutions etc.



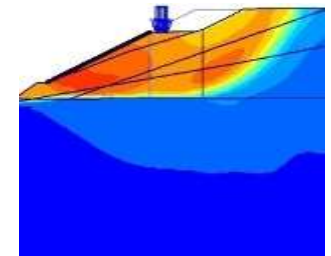
*(potentially embedded in simple programs /
spreadsheets etc.)*

Gap...

More:

- complex
- time consuming
- input parameters
- expertise required
- accurate [potentially at least!]

‘Advanced’:
based on non-
linear finite
elements etc.



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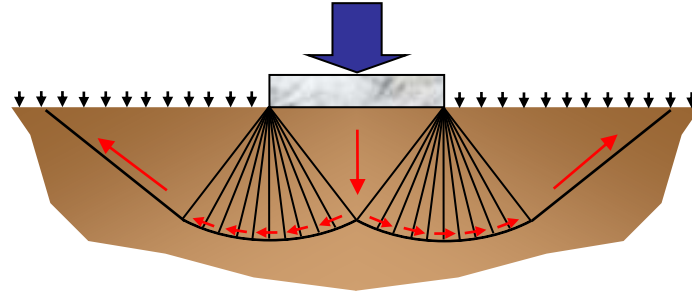
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'Traditional' example



- Bearing capacity
 - Terzaghi equation:

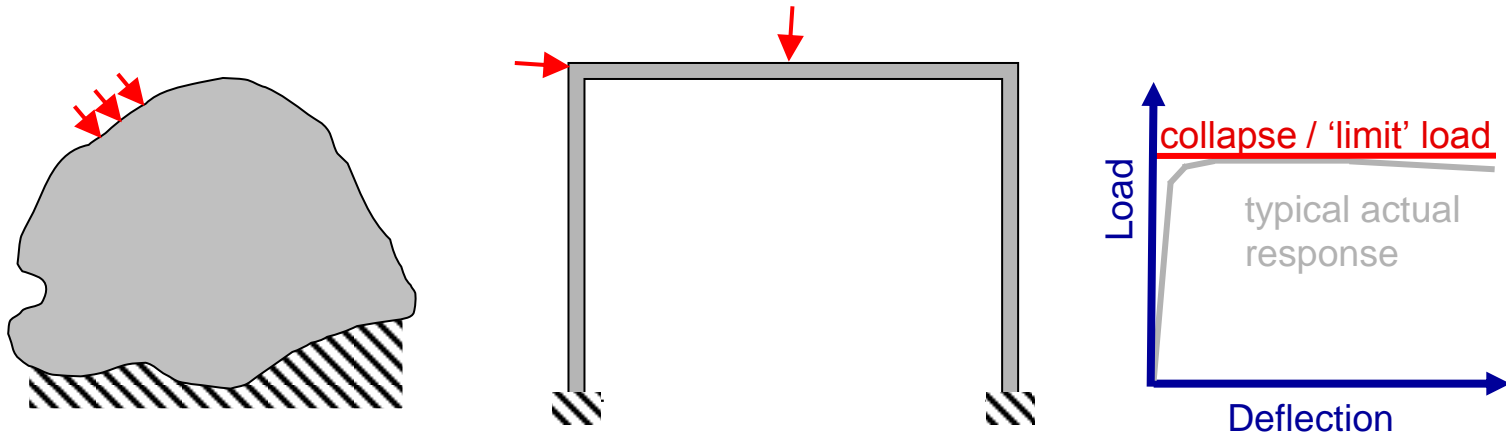
$$q = c s_c d_c i_c N_c + q s_q d_q i_q N_q + \frac{1}{2} \gamma B s_\gamma d_\gamma i_\gamma N_\gamma$$

[Approximate as terms in equation are not really additive – though addition always gives conservative result]

Issue: to handle embedded foundations, inclined loads, layered soils etc. it is necessary to apply additional modification factors. This becomes increasingly imprecise as more factors are used.

The role of limit analysis

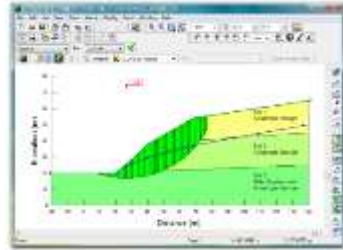
- Many 'traditional' methods use limit analysis
- Limit analysis allows direct estimation of the maximum load sustainable by a body or structure



- **Question:** can limit analysis be computerised?

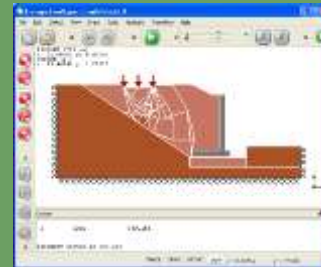
Available geotechnical software

'Traditional':
based on hand
analysis
solutions etc.



*(potentially embedded in simple programs /
spreadsheets etc.)*

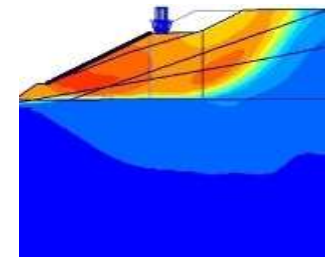
'Mainstream':
based on
computational
limit analysis(?)



More:

- complex
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'Advanced':
based on non-
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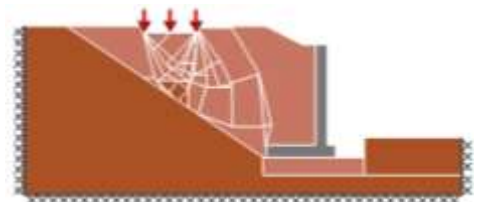
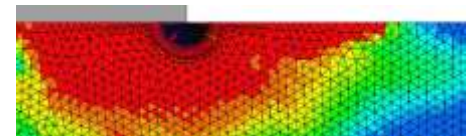
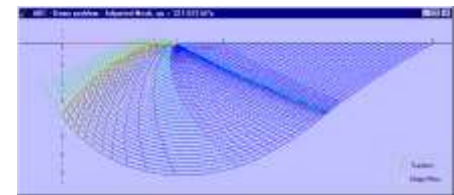
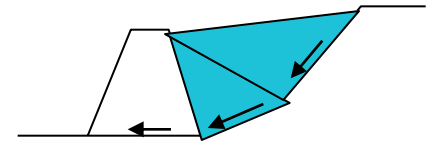
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Computational limit analysis

- Automated single application methods, e.g:
 - Foundations: slip-line field analysis
 - Walls: Coulomb wedge analysis
- Numerical methods
 - Method of characteristics
 - e.g. ABC software for foundations
 - Finite element limit analysis
 - Research tool only at present
 - Discontinuity Layout Optimization (DLO)
 - Now widely used in industry





Material Explorer

- Very Stiff Clay
- Stiff Clay
- Firm Clay
- Soft Clay
- Very Soft Clay
- Dense Sand
- Medium-Dense Sand
- Loose Sand
- Concrete
- Steel
- Weightless Cohesive Soil
- Weightless Concrete
- Frictionless
- No-Tension Cutoff

PROBLEM SIZE (m):
 x: 0.00000 y: 0.00000
 CURSOR (m):
 x: -0.68569 y: 0.57678

New Project

Quick Create

- Empty
- Simple Footing
- Vertically Loaded Footing
- Laterally Loaded Footing
- Gravity Wall
- Stem Wall
- Slope Stability
- Pipeline

Use previously entered values as defaults

OK Cancel

Property Editor

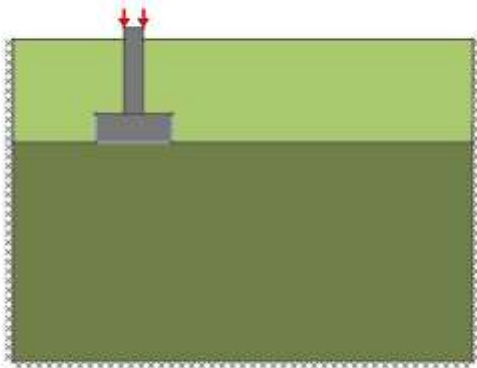
Project

Property	Value
Nodal Density	Medium
Project Name	New Project
Project Details	
Water	Enabled
Long Term Analysis	False
Model Rotations	Along edges
Analysis Options	
Seismic Actions	

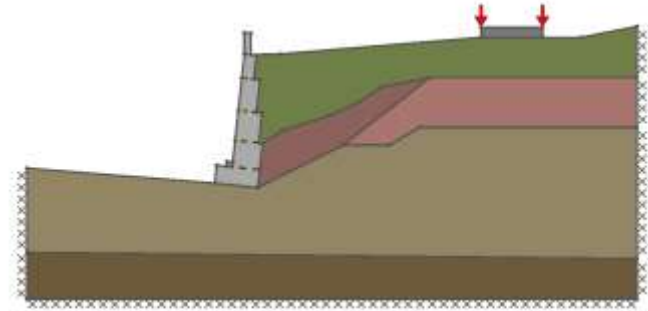
Geometry Editor

x (m)	y (m)

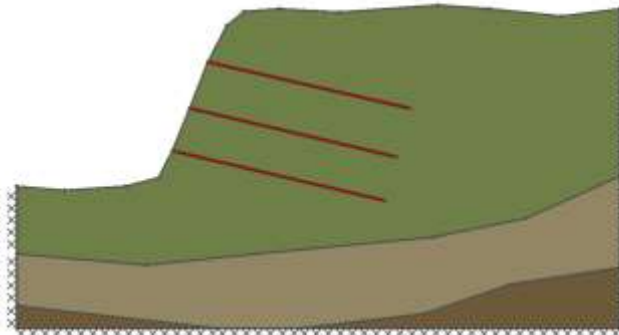
Output



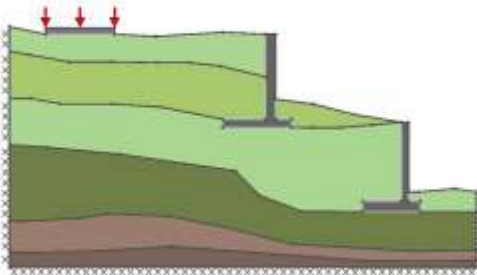
Footings



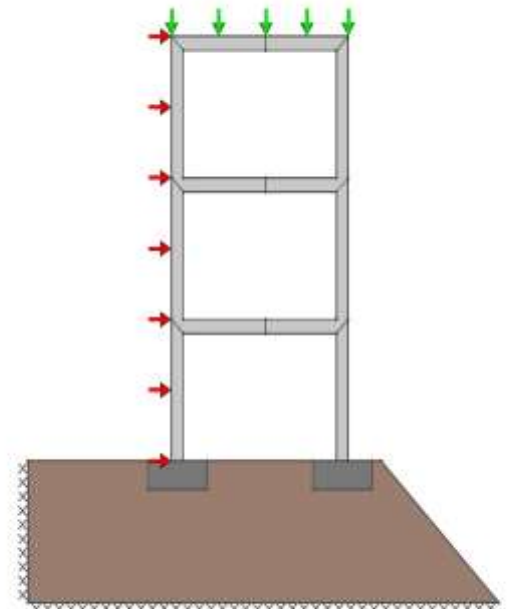
Retaining walls



Slopes



'Combined'



Soil-structure interaction

Discontinuity Layout Optimization (DLO)

Discontinuity Layout Optimization (DLO)

- Can be used to provide rigorous upper bound limit analysis solutions:
 - rapid and **direct** means of analysing the collapse state
 - outcome of 5 year **EPSRC** (UK Government) funded research project carried out at the **University of Sheffield**
 - key publication: Smith & Gilbert, *Proc. Roy. Soc. A*, 2007

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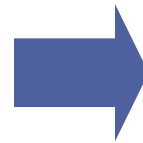
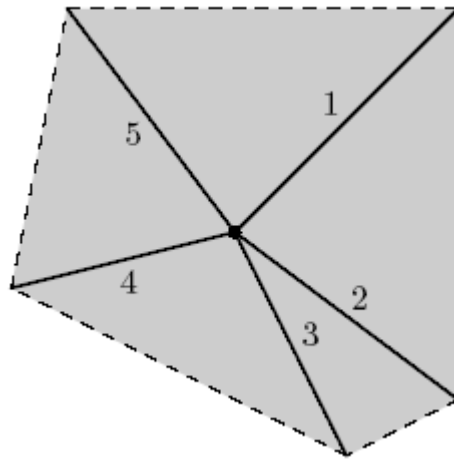
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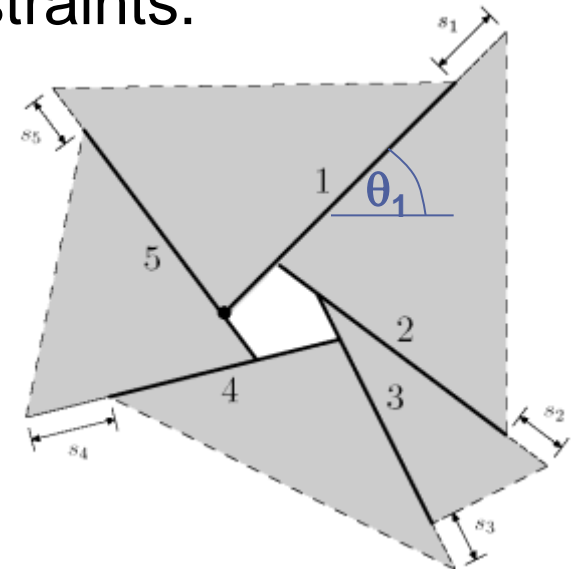
DLO: problem formulation

- Upper bound ('mechanism') formulation
 - Find minimum multiplier on specified loads (margin of safety, or 'adequacy factor')
 - Subject to nodal compatibility constraints:



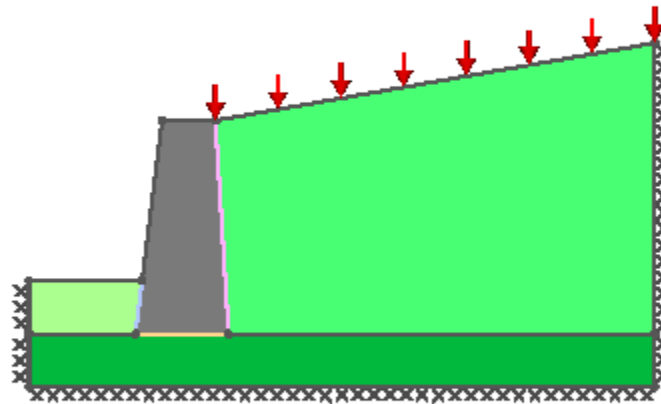
$$\sum_{i=1}^5 s_i \cos \theta_i = 0$$

$$\sum_{i=1}^5 s_i \sin \theta_i = 0$$



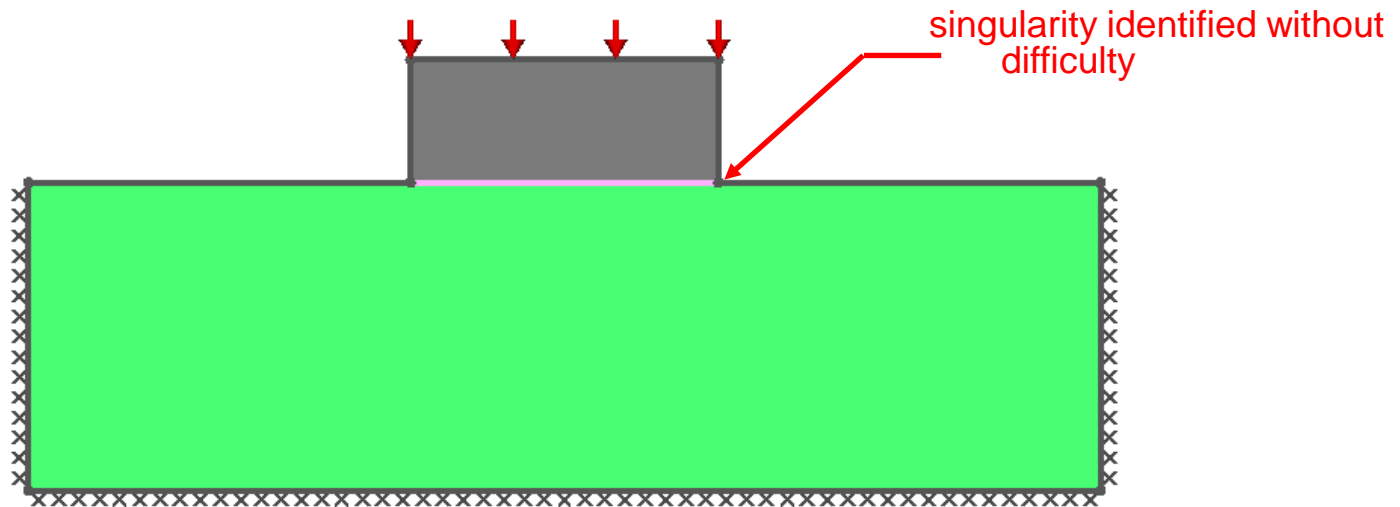
DLO: obtaining a solution

- Problem is a simple linear optimization problem (easy to solve)
- After solving can identify and then deform solid blocks to help interpretation:



Application to benchmark problems

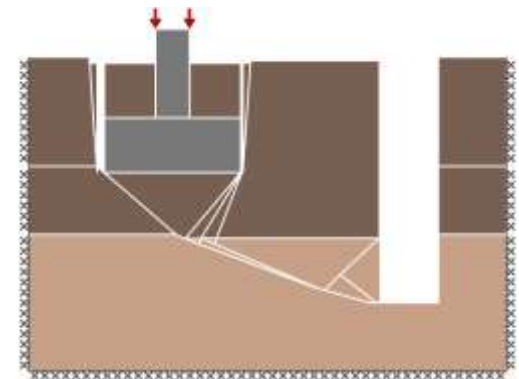
- For 'Prandtl punch' problem, solution within 1% of exact solution $(2+\pi)$ in approx. 1 second:



- Results for 100+ other cohesive-frictional benchmark plane strain problems available at: <http://www.limitstate.com/geo/verification>

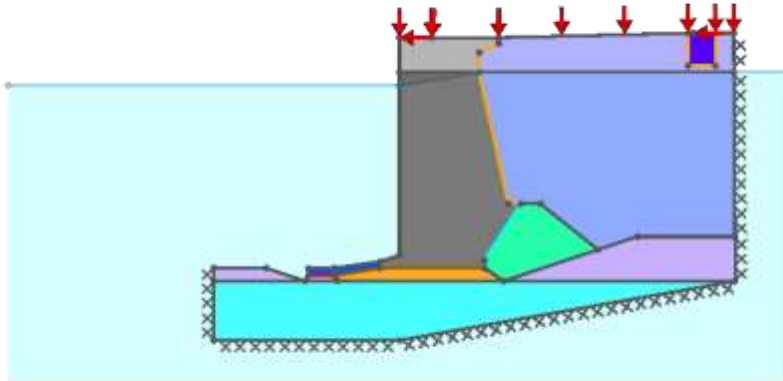
Application examples

Inclined footing, carrying stadium roof loads

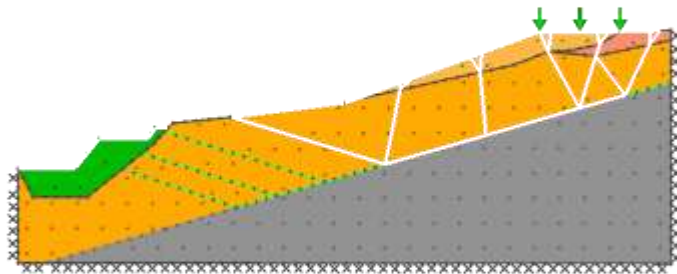


Trench excavation near footing

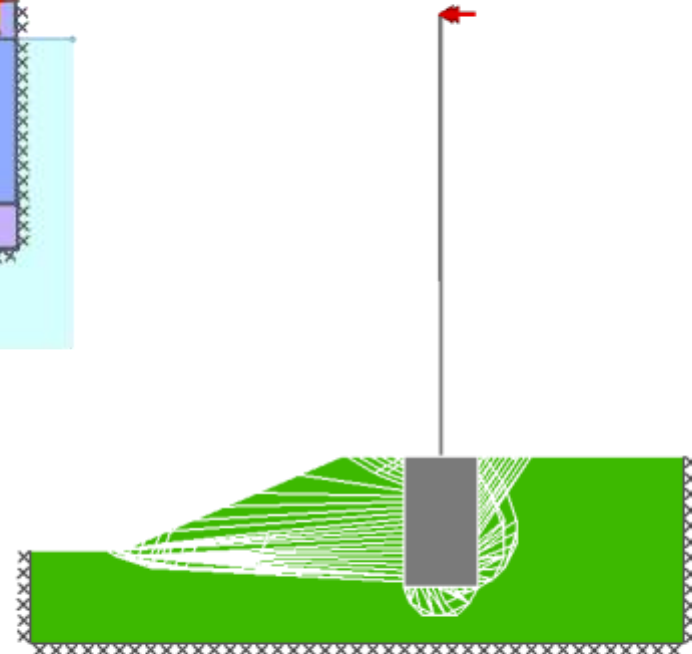
Application examples



Quay wall

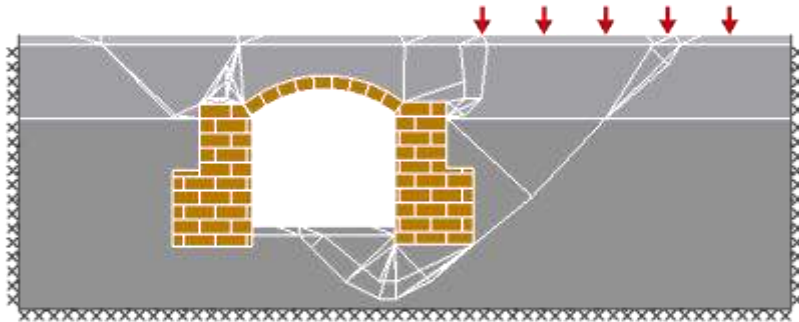


Slope remediation

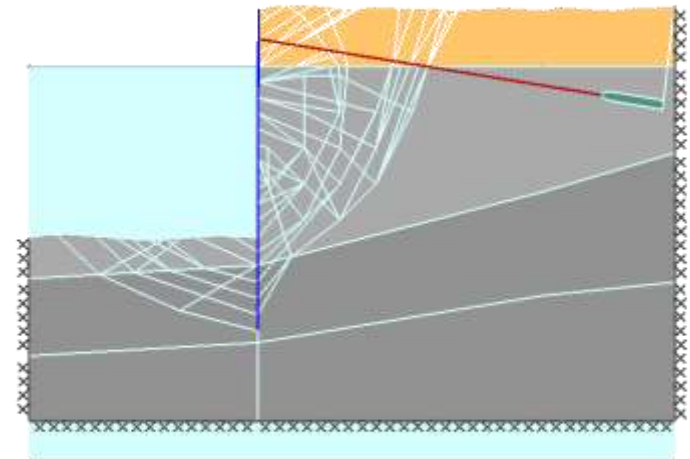


Mast foundation

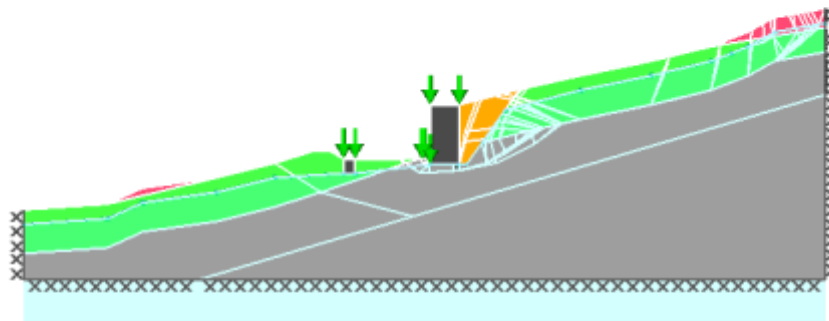
Application examples



Masonry arch stability



Anchored sheet pile wall



Construction on slope

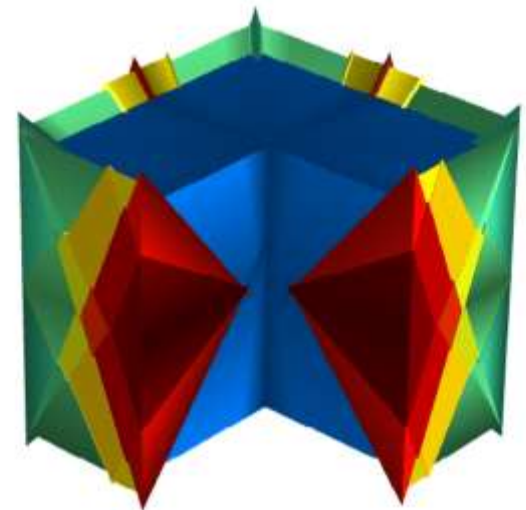
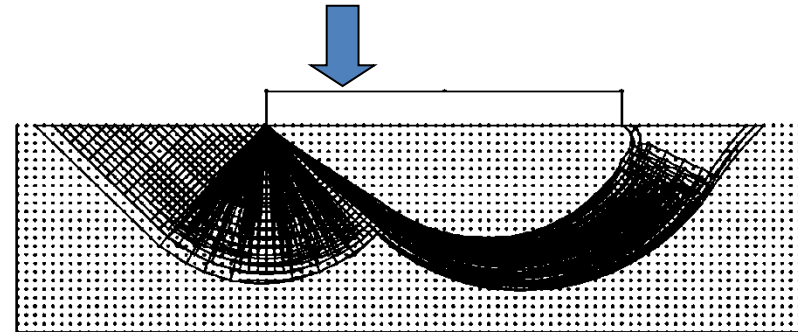
Application in practice

- Once embedded in a modern software application, methods such as DLO takes the work out of analysis and allow the engineer to focus on:
 - Interpreting and understanding the ground conditions
 - Developing innovative designs
 - What if/parametric studies



Recent academic developments

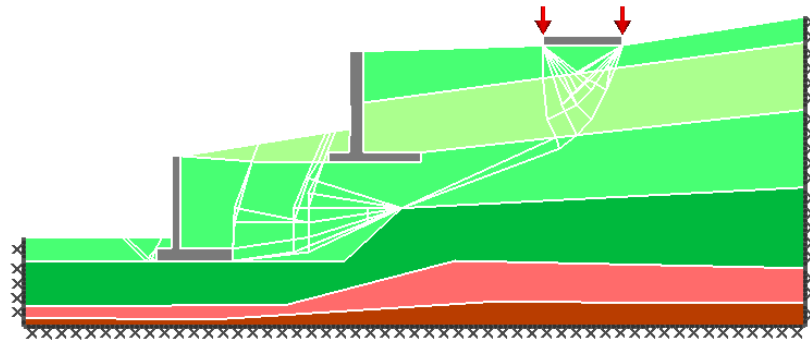
- Curved slip-lines to model rotational failures
 - Smith & Gilbert, *Géotechnique*, November 2013
- 3D DLO
 - Hawksbee, Smith & Gilbert, *Proc. Roy. Soc. A*, July 2013



Application in design

Application in design

- DLO provides engineers with a fully **general-purpose** *direct* ULS analysis capability.

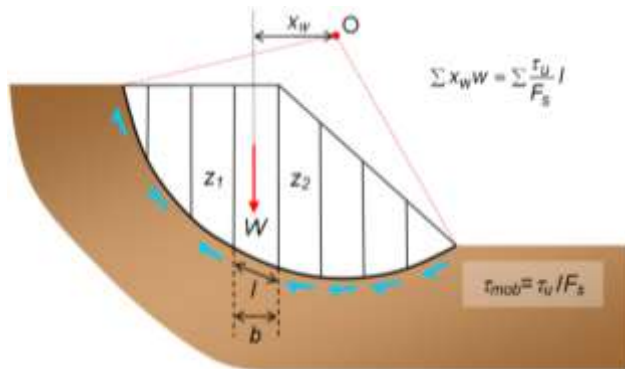


- How can this be used with existing and new design codes, including limit state codes such as Eurocode 7?

Slope stability

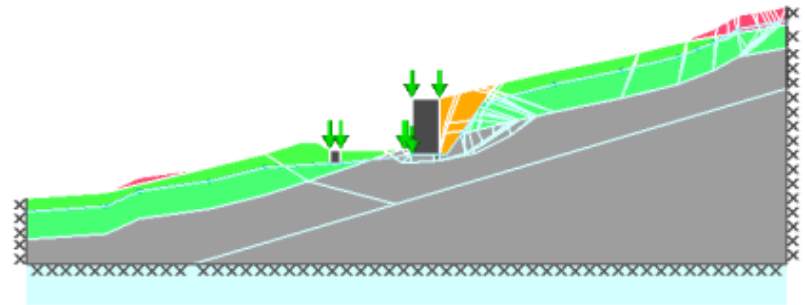
Conventional approach:

- Factor on strength averaged over length of **assumed** slip surface.



General approach:

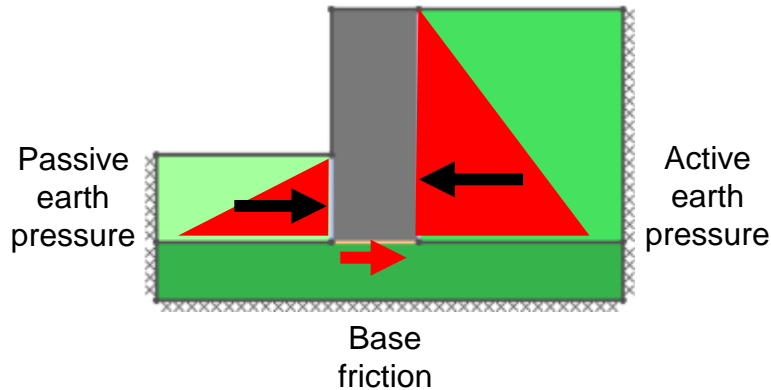
- Factor on strength applied to whole soil domain. Software **automatically determines** critical failure mechanism.



Retaining walls

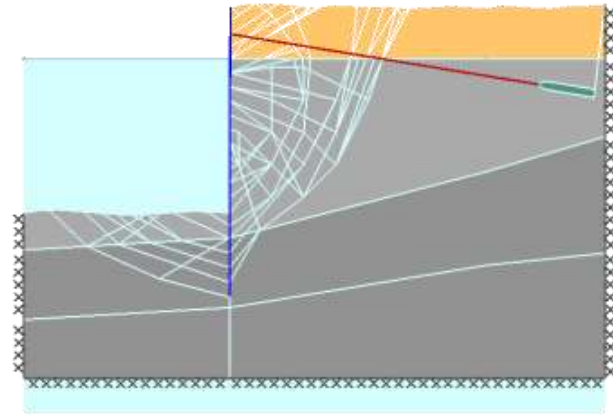
Conventional approach:

- Active and passive earth pressures are **assumed** to act on the wall. K_a and K_p determined from factored soil strength.



General approach:

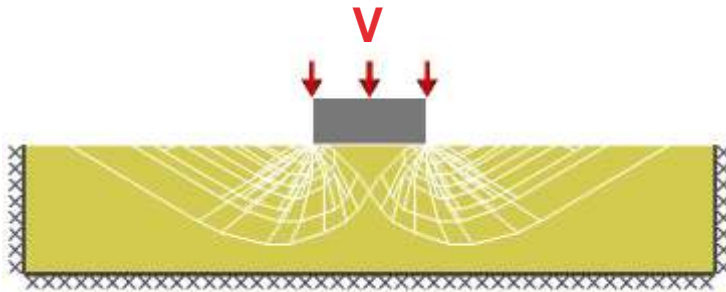
- Factor on strength applied to whole soil domain. Software **automatically determines** critical failure mechanism.



Foundations

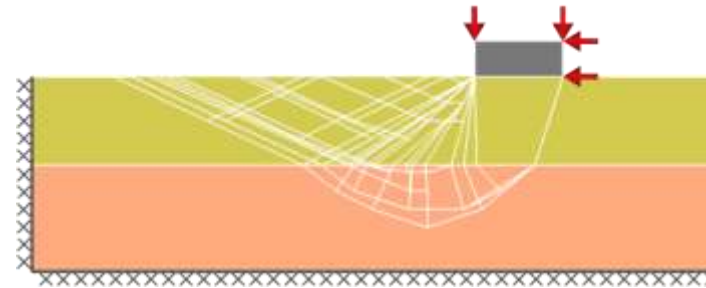
Conventional approach:

- Determine collapse load using Terzaghi's bearing capacity equation (implicitly **assuming** a specific collapse mode). Factor load by 3.



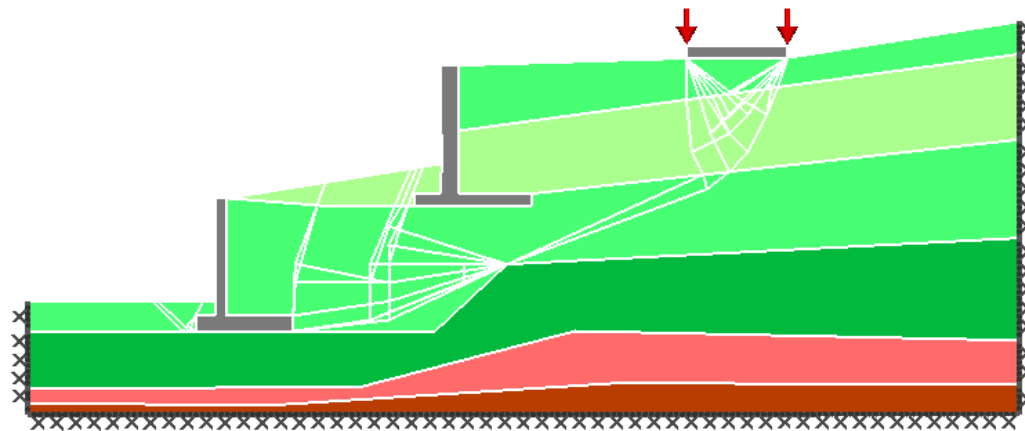
General approach:

- Determine collapse load from **automatically determined** critical failure mechanism. Factor load by 3.



What about limit state design?

- Limit state design codes such as Eurocode 7 (EC7) are written in a general way, often tying in with **general purpose** analysis approaches



- e.g. EC7 Design Approach 1 (DA1) Combination 2 allows direct use of **general purpose** numerical limit analysis

What about limit state design? [2]

- Before undertaking an analysis **partial factors** are applied to loads and/or materials
- An **adequacy factor** is then applied to load(s) or material strength(s) to trigger collapse:
 - If the **adequacy factor** ≥ 1.0 then the system is stable.
 - If the **adequacy factor** < 1.0 then the system is unstable.

Middle East examples

Example 1: embankment on soft soil

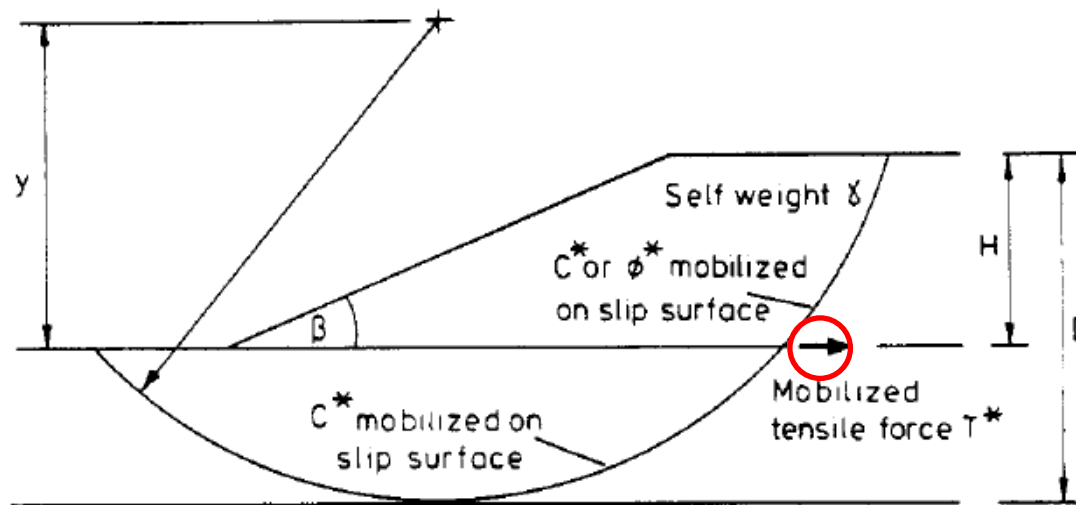
- Soft soils are common in the region:



(Photos from Benmebarek et al., TAJSAT, 2013)

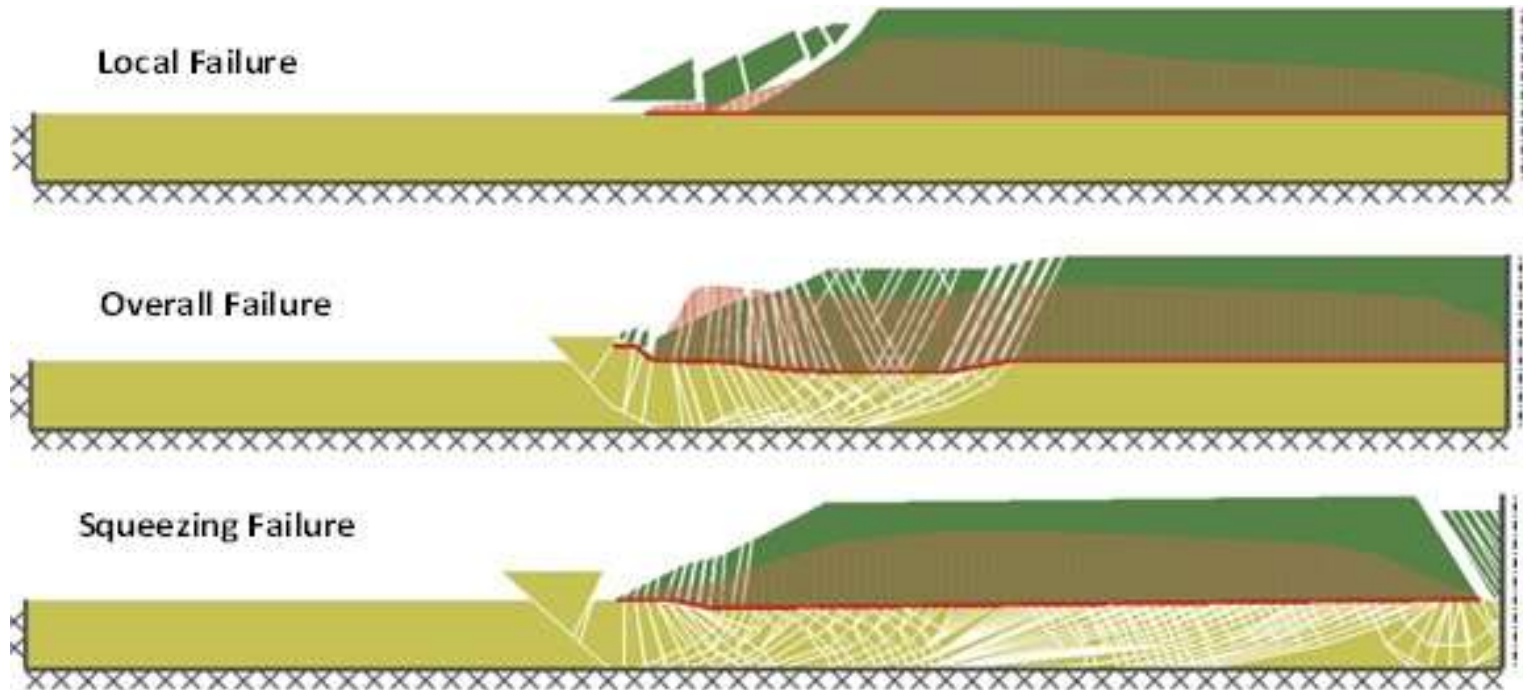
'Traditional' model

- Traditional methods of analysis rely on assumed mechanisms
- e.g. Hird (1986):



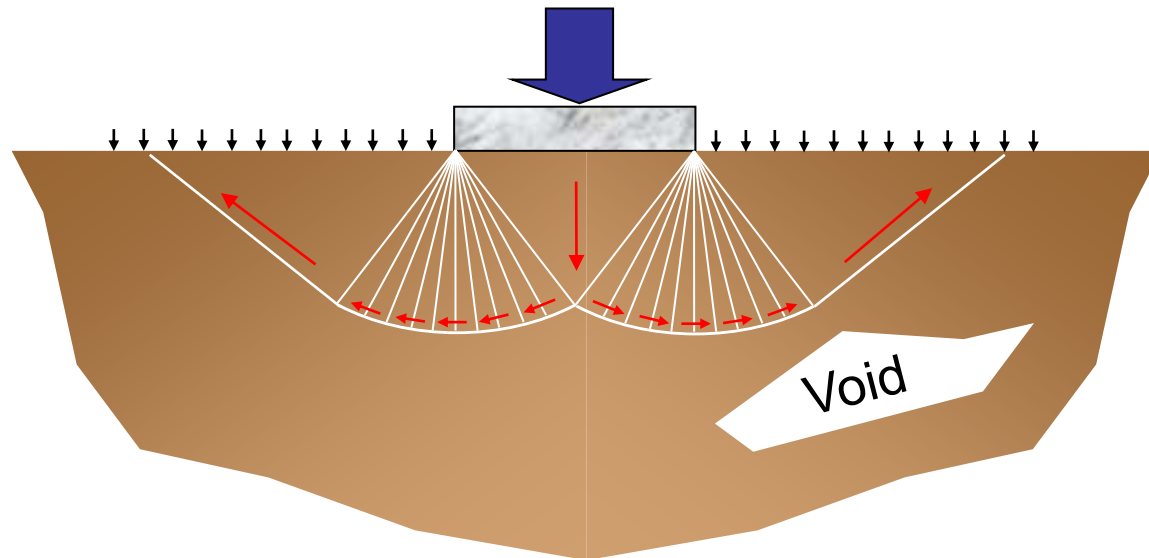
LimitState:GEO model

- Key point: able to identify a wider range of mechanisms than traditional methods, e.g:



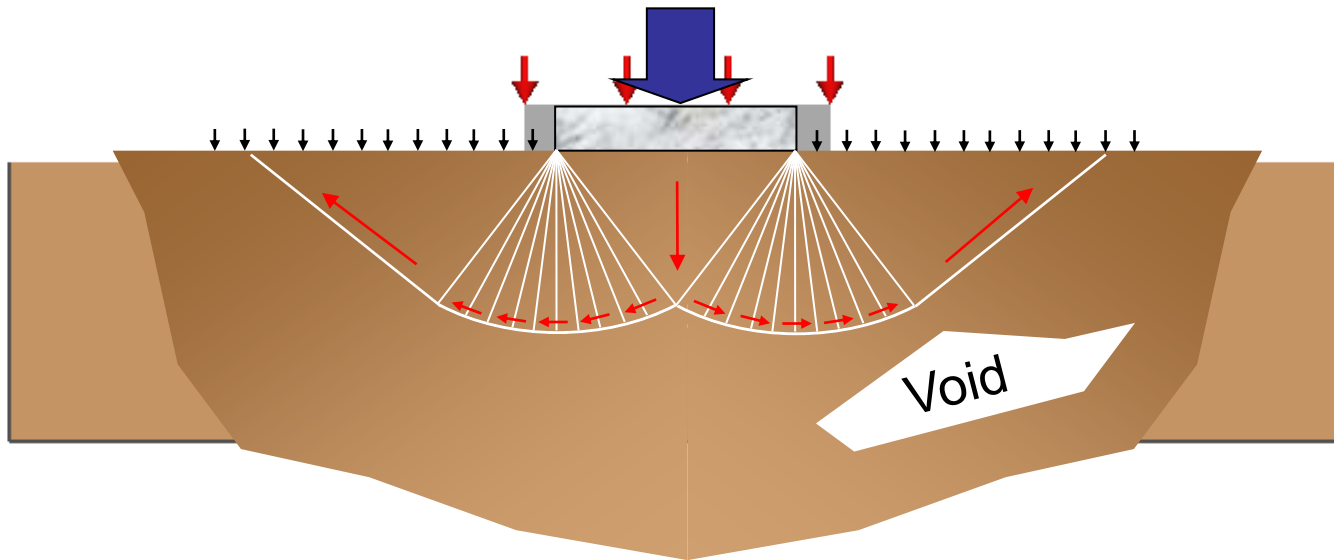
Example 2: foundation close to cavity

- Dissolution cavities common in the region



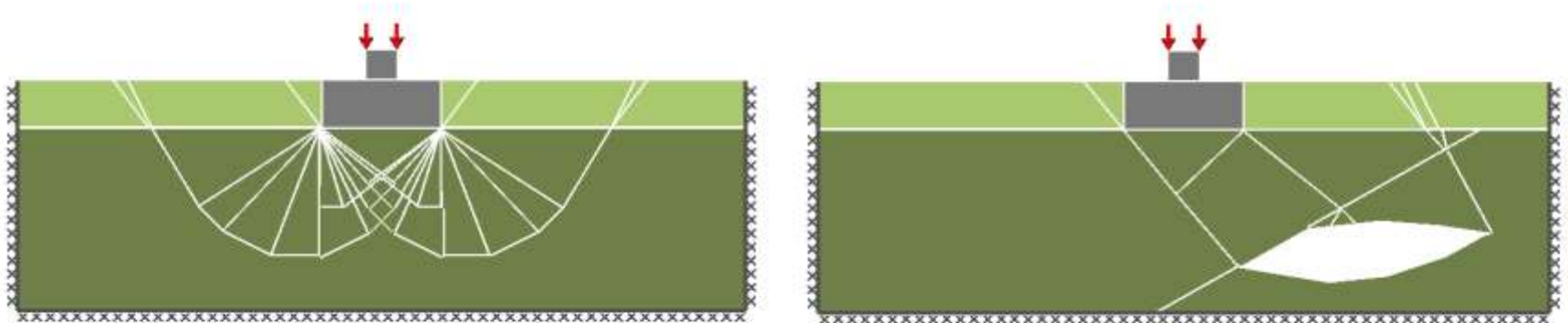
- Problem: difficult to model these using traditional methods!

Example 2: foundation close to cavity



LimitState:GEO model

- Easy to model effects of arbitrary features, such as voids



(assuming their location & extent are known!)

Other applications

- Modelling of soil improvement methods
 - Cut and fill
 - Stone columns
 - Ground mixing
 - Etc, etc...



Conclusions

Conclusions

- Computational limit analysis fills the gap between ‘traditional’ and ‘advanced’ tools:
 - Generally applicable methods fit in well with the philosophy of limit state codes such as Eurocode 7
- DLO is a quick and easy to use general method:
 - Automatically identifies the critical mechanism
 - Eliminates the need to separately consider sliding, bearing failure etc, or to try to fit a problem to a known solution
- Flexibility to model non-standard features / geometries is important for many Middle East applications

Acknowledgements

- LimitState credits:
 - Dr Wael Darwich, Dr Tom Pritchard
- University of Sheffield credits:
 - Dr Colin Smith, Dr Sam Hawksbee, Alireza Tatari, Ahmed Babiker
 - EPSRC (UK Government Research Council)